On two-atom scattering in a quantized radiation field

1976 J. Phys. A: Math. Gen. 9 225

(http://iopscience.iop.org/0305-4470/9/2/006)

J. Phys. A: Math. Gen., Vol. 9, No. 2, 1976. Printed in Great Britain. © 1976

On two-atom scattering in a quantized radiation field

E Prugovečki† and A Tip‡

†Department of Mathematics, University of Toronto, Toronto M5S 1A1, Canada
‡FOM-Instituut voor Atoom- en Molecuulfysica, Kruislaan 407, Amsterdam/Wgm, The Netherlands

Received 4 July 1975, in final form 29 October 1975

Abstract. For the scattering of a pair of model atoms in the presence of a quantized radiation field, consisting of a finite number of field modes, the existence of the wave operators $\Omega_{\pm}$ is proven. This supplements our earlier results where an external classical radiation field was considered.

1. Introduction

In two recent papers (Prugovečki and Tip 1974a, b, to be referred to as I and II respectively), we considered the scattering of two atoms (strictly, two neutral particles with internal structure) in the presence of an external classical radiation field. Under these circumstances, the Hamiltonian of the system is time-dependent and this fact gives rise to various complications as compared to the time-independent case.

Nevertheless, we were able to demonstrate in I and II that Møller wave operators still exist under reasonable conditions on the interaction potential and the field configuration. On the other hand, it turns out that this approach leads to a number of complications if one wants to set up a perturbation scheme. This is due to the time-dependence of the Hamiltonian which gives rise to secular behaviour. Specific perturbation techniques such as the multiple-time-scale method or the Magnus expansion then become necessary.

An atomic system coupled to a quantized radiation field, however, can be described by means of a time-independent Hamiltonian and consequently perturbation techniques formulated within the framework of the stationary method now become available. We therefore investigated whether the results on the existence of wave operators, obtained in I and II, carry over to this situation. As shown in the remainder of this paper, this indeed turns out to be the case under the conditions on the interaction potentials adopted in I and II, provided the atoms are coupled to a finite number of spatially localized field modes. In § 2 we give a precise description of the proposed model, whereas the actual existence proof for the wave operators can be found in § 3.

2. The Hamiltonian of the model

We consider two particles with a finite or countably infinite number of internal states and assume that they collide in the presence of a radiation field with a finite number N of modes. Let H_j denote the free Hamiltonian of the jth particle, $j = 1, 2$. It consists of a translational part $H_j^{tr} = p_j^2/2m_j$ and an internal part H_j^{int}:

$$H_j = H_j^{tr} + H_j^{int}. \tag{2.1}$$

Here $\boldsymbol{p}_j$ is the momentum operator for particle j and m is its mass. H_j^{int} has a finite or countably infinite discrete spectrum, for details see I-§ 1 and II-§ 3. As before, we represent H_j^{int} as diagonal operator $(H_j^{int})_{\alpha\beta} = \omega_\alpha(j)\delta_{\alpha\beta}$ on a suitable l^2-space.

The free radiation Hamiltonian reads (in units with $\hbar = c = 1$):

$$H_{rad} = \sum_{\nu=1}^{N} k_\nu b_\nu^\dagger b_\nu \tag{2.2}$$

where $b_\nu^\dagger$ and b_ν are the creation and annihilation operators for the νth radiation mode.

Since we use a point-particle model for the colliding atoms, it is consistent to use an interaction Hamiltonian of particle j with the field of the form (Stenholm 1973)

$$V_{j,rad} = (2\epsilon_0)^{-1/2}\boldsymbol{\mu}_j \cdot \sum_{\nu=1}^{N} k_\nu^{1/2}\, \boldsymbol{U}_\nu^e(\boldsymbol{x}_j)(b_\nu + b_\nu^\dagger). \tag{2.3}$$

Here $\boldsymbol{\mu}_j$ is the dipole-moment operator of particle j, which acts in the internal state space of this particle, whereas ϵ_0 is the dielectric constant. Thus we can characterize $\boldsymbol{\mu}_j$ by means of the matrix $\boldsymbol{\mu}_{j,\alpha\beta}$. As in I and II, we assume $\boldsymbol{\mu}_j$ to be bounded. The functions $\boldsymbol{U}_\nu^e(\boldsymbol{x}_j)$, where $\boldsymbol{x}_j$ is the position operator of particle j, are those discussed by Stenholm (Stenholm 1973). Thus they are square integrable functions with the orthogonality property

$$\int \mathrm{d}\boldsymbol{x}\, \boldsymbol{U}_\nu(\boldsymbol{x}) \cdot \boldsymbol{U}_{\nu'}(\boldsymbol{x}) = \delta_{\nu\nu'}. \tag{2.4}$$

The interaction-free Hamiltonian is now given by

$$H_0 = H_1 + H_2 + H_{rad} \tag{2.5}$$

which acts in the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{F}$ where $\mathcal{H}_j$ is the Hilbert space for particle j and $\mathcal{F}$ the Fock space for the considered modes of radiation. The full Hamiltonian now reads

$$H = H_0 + V_{12} + V_{1,rad} + V_{2,rad} = H_0 + V \tag{2.6}$$

where $V_{12} = V_{12}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$ represents the interaction between the two particles. Under the conditions imposed upon V_{12} in II and with the present form of $V_{j,rad}$, H is self-adjoint on $\mathcal{H}$.

3. The existence of wave operators

We show now that the wave operators

$$\Omega_\pm = s - \lim_{t\to\pm\infty} \exp(\mathrm{i}Ht)\exp(-\mathrm{i}H_0 t) \tag{3.1}$$

exist under the conditions on the interaction terms discussed in § 2. This is achieved by establishing the existence of a set $\mathcal{D}$, dense in $\mathcal{H}$, consisting of state vectors Ψ for which

$$\int_{-\infty}^{+\infty} \mathrm{d}t \|V\exp(-\mathrm{i}H_0 t)\Psi\|$$
$$\leqslant \sum_{j=1}^{2}\int_{-\infty}^{+\infty} \mathrm{d}t\|V_{j,rad}\exp(-\mathrm{i}H_0 t)\Psi\| + \int_{-\infty}^{+\infty} \mathrm{d}t\|V_{12}\exp(-\mathrm{i}H_0 t)\Psi\| < \infty. \tag{3.2}$$

The general element of $\mathscr{D}$ is taken to be a finite linear combination of states of the form

$$\Psi = \psi(\boldsymbol{x}_1, \boldsymbol{x}_2) \otimes |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes |\boldsymbol{n}\rangle \tag{3.3}$$

where $|\alpha_j\rangle$ is an eigenstate of H_j^{int} at the energy ω_{α_j}, while $|\boldsymbol{n}\rangle = |n_1\rangle \ldots |n_N\rangle$ is the eigenstate of the field Hamiltonian with exactly n_ν photons in the mode ν. We note (with $\boldsymbol{n}.\boldsymbol{k} = \sum_{\nu=1}^{N} n_\nu \boldsymbol{k}_\nu$) that

$$\exp(-H_0 t)\Psi = \exp[-\mathrm{i}(\omega_{\alpha_1} + \omega_{\alpha_2} + \boldsymbol{n}.\boldsymbol{k})t]\Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t) \otimes |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes |\boldsymbol{n}\rangle \tag{3.4}$$

where the time-evolution of the translational part $\Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t)$ of the wavefunction follows from that of its Fourier transform $\tilde{\Psi}(\boldsymbol{p}_1, \boldsymbol{p}_2)$:

$$\tilde{\Psi}(\boldsymbol{p}_1, \boldsymbol{p}_2, t) = \exp[-\mathrm{i}(\boldsymbol{p}_1^2/2m_1 + \boldsymbol{p}_2^2/2m_2)t]\tilde{\psi}(\boldsymbol{p}_1, \boldsymbol{p}_2). \tag{3.5}$$

In order to proceed, we restrict the class of functions $\Psi(\boldsymbol{x}_1, \boldsymbol{x}_2)$ to those contained in $\mathscr{D}_1 \otimes \mathscr{D}_2$, $\mathscr{D}_j$ being defined in II-lemma 3.1 (with $\boldsymbol{k}$ replaced by $\boldsymbol{p}$). We observe that this restriction still guarantees that $\mathscr{D}$ is dense in $\mathscr{H}$. Under the conditions imposed on V_{12}, it follows, by using the method outlined in II-§ 3, (with G and ϵ being positive constants) that

$$\|V_{12}\exp(-\mathrm{i}H_0 t)\Psi\| \leq G(1 + 4t^2)^{-\frac{1}{2}-\frac{1}{4}\epsilon}$$

since the presence of the quantized radiation field does not alter the estimates for this term.

For the other terms in equations (3.2) we have

$$\|V_{j\text{rad}}\exp(-\mathrm{i}H_0 t)\Psi\|^2 = \sum_{\nu=1}^{N} [(2n_\nu + 1(k_\nu/2\epsilon_0]\langle\alpha_j|\boldsymbol{\mu}_j\boldsymbol{\mu}_j|\alpha_j\rangle : \int \mathrm{d}\boldsymbol{x}_1\, \mathrm{d}\boldsymbol{x}_2\, \boldsymbol{U}_\nu^{\mathrm{e}}(\boldsymbol{x}_j)\boldsymbol{U}_\nu^{\mathrm{e}}(\boldsymbol{x}_j)|\Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, t)|^2 \tag{3.6}$$

where the colon indicates a double contraction over tensorial indices ($\boldsymbol{\mu}_j$ and $\boldsymbol{U}_\nu^{\mathrm{e}}$ are vectors). Since $\boldsymbol{U}_\nu^{\mathrm{e}}(\boldsymbol{x})$ is square integrable, it follows that equation (3.6) decreases faster than t^{-2} for large t and consequently the finiteness of the remaining terms in equation (3.2) is proven.

Thus the wave operators $\Omega_\pm$ exist for the case at hand. The assumption that the number of field modes is finite is essential, since infrared and ultraviolet divergencies will make their appearance otherwise. This means that the present results correspond to the case of multiperiodic fields in the semi-classical case. In order to deal with fields with a continuous frequency spectrum over a limited frequency range, equation (2.3) should be replaced by a particle-field interaction term containing a cut-off function for small and large photon energies.

Acknowledgments

EP was supported in part by a grant from the National Research Council of Canada. AT was supported by the Foundation for Fundamental Research on Matter (FOM), which is sponsored by the Netherlands Organization for the Advancement of Pure Research (ZWO).

References

Prugovečki E and Tip A 1974a *J. Phys. A: Math., Nucl. Gen.* **7** 572–85
—— 1974b *J. Phys. A: Math., Nucl. Gen.* **7** 586–96
Stenholm S 1973 *Phys. Lett.* **6C** 1–122